On Guillotine Separable Packings for the Two-dimensional Geometric Knapsack Problem

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Joint work with:

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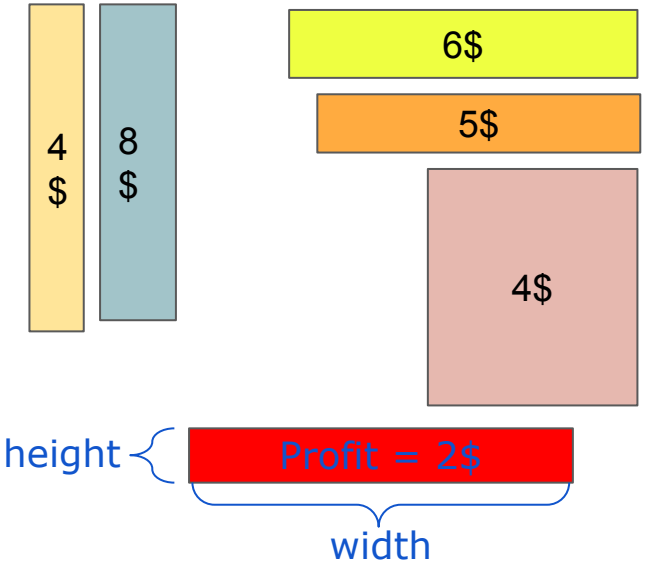
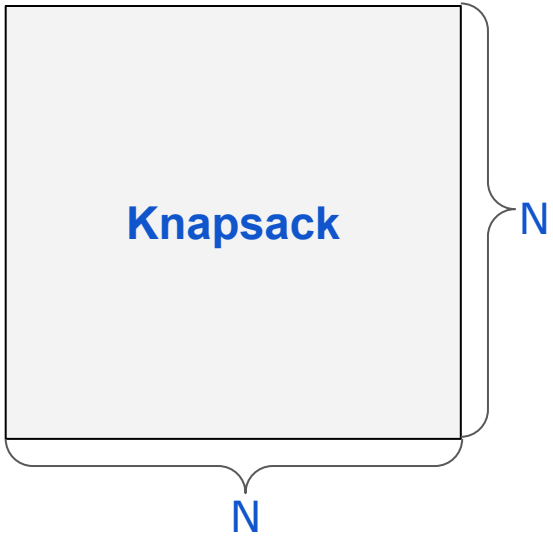
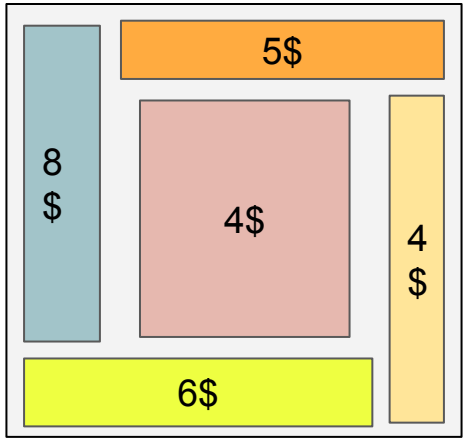
2D Geometric Knapsack

- Input:
  - Items : Axis Parallel Rectangles
  - Knapsack : N x N Square
- Goal : Pack most profitable non-overlapping subset of items





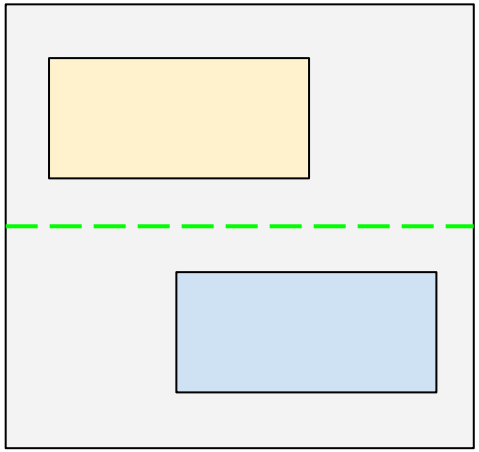


OPT = 27$

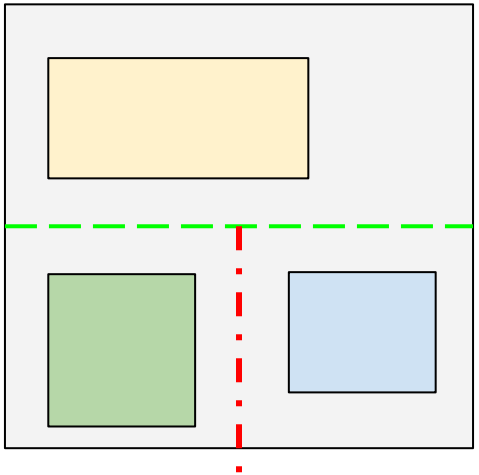
Guillotine Separability

Each item can be "cut" out using axis parallel end-to-end cuts

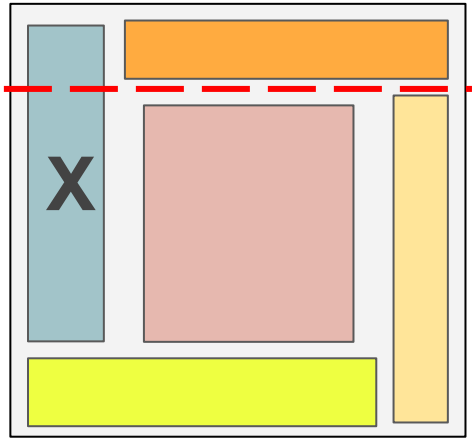
Guillotine Separable



Guillotine Separable

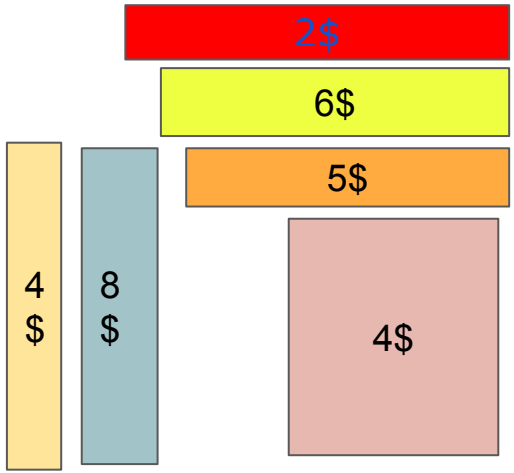


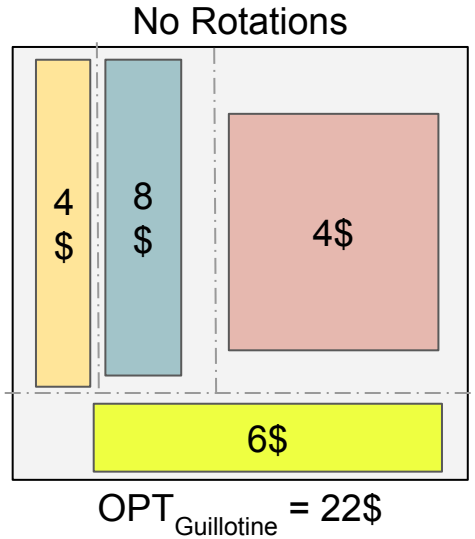
Not Guillotine Separable

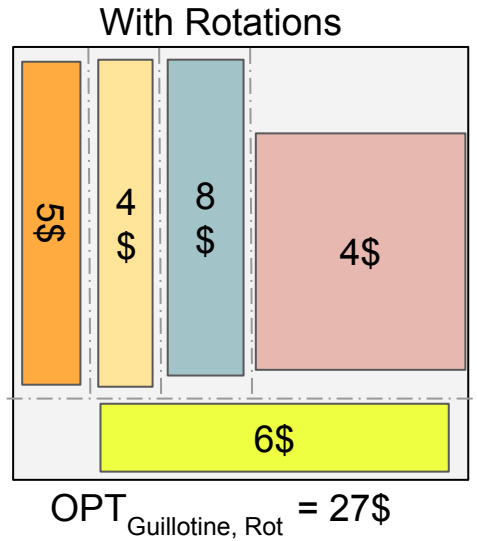


2D Guillotine Knapsack

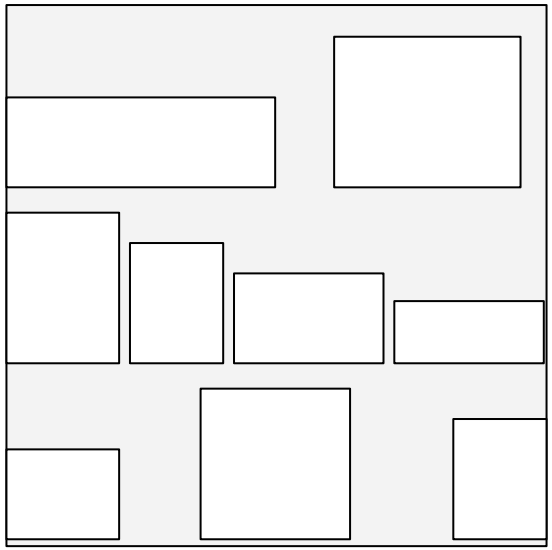
- Input
  - Knapsack : N x N Square
  - Items : Axis Parallel Rectangles
- Goal : Pack most profitable subset of items such that items are
  - Non-Overlapping
  - Guillotine Separable



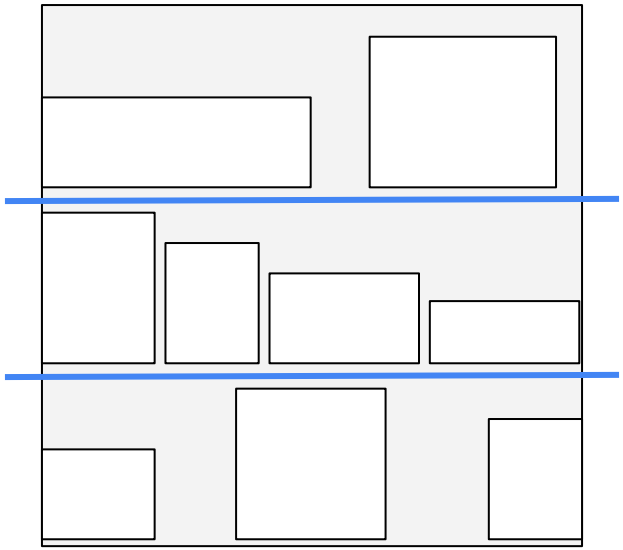




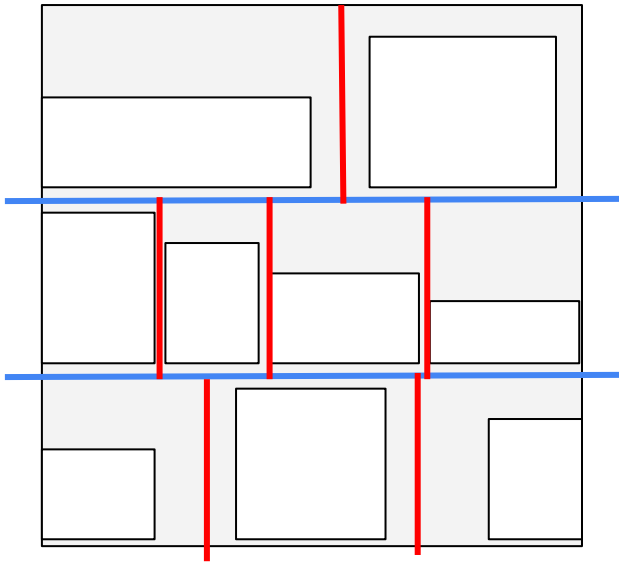
A Guillotine Separable Packing



Packing



First Stage Cuts



Second Stage Cuts

Variants & Prior Results

- Variants
  - Rotated Case: Items can be rotated by 90°
  - Cardinality Case: All Profits = 1

- NP-Hard
- (3 + ε)-approximation [Jansen and Zhang, SODA'04]
- Cardinality Case: QPTAS with quasi-polynomially bounded input [Abed et al, Approx'15]

Our Results

- Pseudo-Polynomial Time Approximation Scheme (PPTAS): $(1 + \varepsilon)$-approximation with pseudo-polynomial running time.

- Input numbers are all polynomially bounded in n

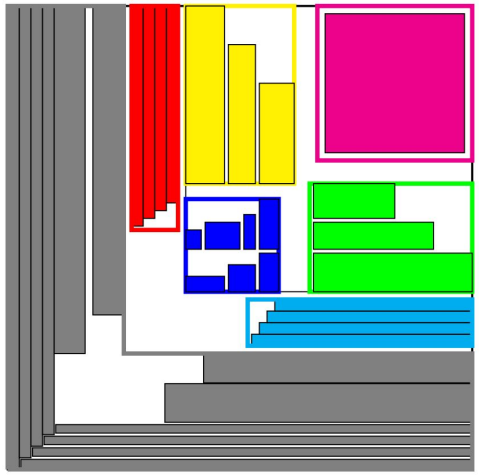
Our Techniques

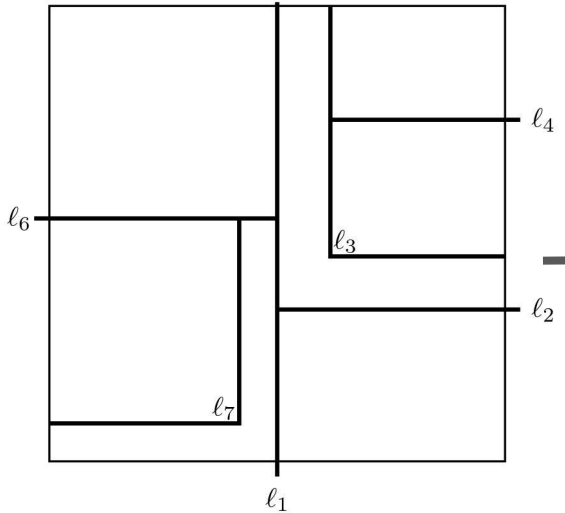
- **Structural Lemma:** Existence of near-optimal "nicely" structured solutions

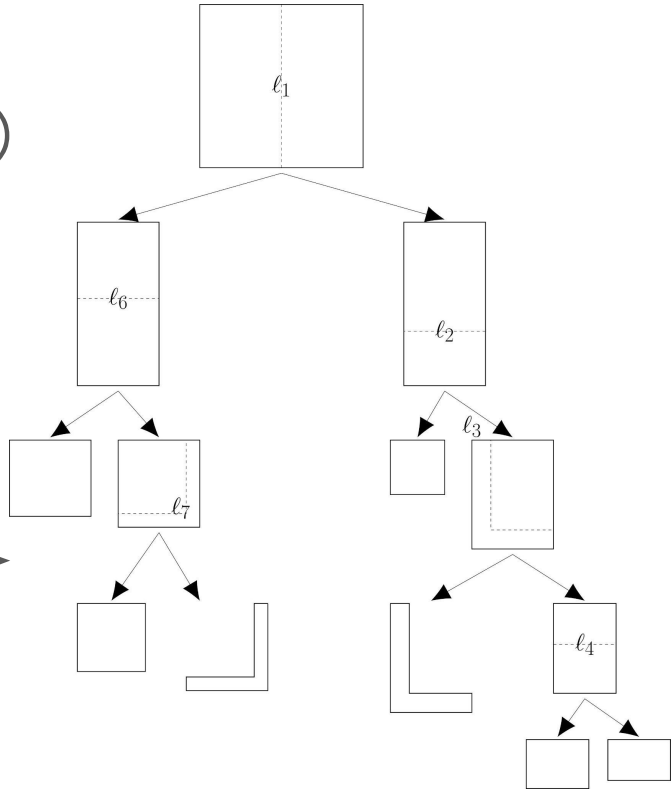
- **Guessing the Packing:** Guess the nice packing in $(nN)^{O_\varepsilon(1)}$

Structural Lemma

- Existence of packing in:
  - $O_\varepsilon(1)$ L or box pseudo-guillotine separable Compartments
  - Near-optimal (1+ε - approximate)
  - Nicely packed

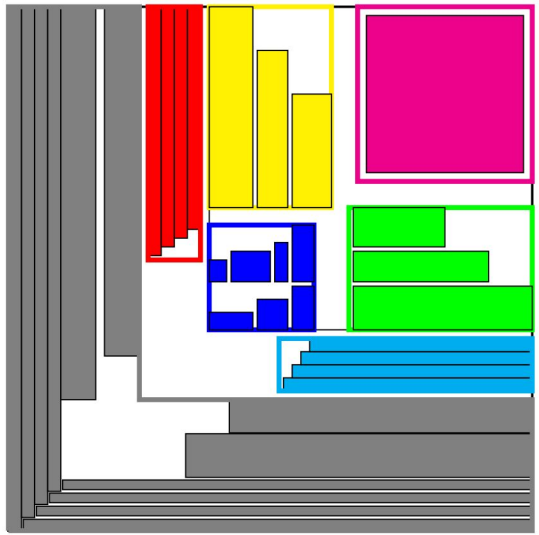






Guessing Near-Optimal Packing

- Guess $O_\varepsilon(1)$ L- or box- Compartments
- Assign & Pack Items
    - box-Compartment : Use GAP -> NFDH
    - L-Compartment : Use algorithm adapted from recent work by Galvez et al (SoCG'21)



Can be extended to case where 90° rotations are allowed

Thank You!!

Questions?

There will be a detailed talk next week in SoCG’21